TO KNOW OR NOT TO KNOW:
Strategic Inattention and Endogenous Market Structure

by
Roberto CELLINI(University of Catania)
Luca LAMBERTINI (University of Bologna)
Gianmarco OTTAVIANO (London School of Econ.)

DSE UNIBO working paper 987

cellini@unict.it
http://www.robertocellini.it

Aims

- To study a firm's choice of whether or not to consider information concerning interdependence.
- Any firm can strategically choose to consider or not the fact that the market total output is affected by its own production choice:
  - if such information is considered, the firm behaves as an oligopolist;
  - if not, the firm behaves in a monopolistically competitive way.
- The market regime is endogenously determined.
- Which behaviour is individually (and socially) optimal? Can different behaviours co-exist?

(Textbook) difference between oligopolistic and monopolistically competitve behaviour

- From a theoretical point of view, the distinction is clear:
  - a MC firm takes for given the aggregate market output when setting the individual production
  - (the aggregate market output is a "parameter", and each MC firm focuses on its niche)
  - oligopolistic firms are aware of their role in determining the market output
  - (oligopolistic firms explicitly consider the effect that an individual choice exerts on the market supply.)

In this paper:

- We study the case in which firms can strategically choose whether or not to consider the effect of their individual decisions on the market total output, …
- … so that the market regime emerges as endogenous (due to the firms' strategic choices).
- Different outcomes can emerge, depending on the number of firms, the product substitutability and the cost structure. Three cases are possible:
  - *There is only one equilibrium, in which all firms are oligopolistic*
  - *There is only one equilibrium, in which all firms are MC*
  - *There are two equilibria, all firms O and all firms MC*

In this paper:

- The information concerning other firms' behaviour is freely accessible (costless)…
- … but, in some circumstances, it could be individually convenient to ignore such information
- Two interpretations:
  - ○ the firm could find it optimal to constrain itself to neglect such available information.
  - ○ the value of certain information can be negative.

Literature discussing the value of information in games, where players rationally prefer to ignore some potentially relevant pieces of information (Bassan, Scarsini, Zamir, 1997; Bassan, Gossner, Scarsini, Zamir, 2003; Kamien, Tauman, Zamir, 1990).

In this paper:

- The information concerning other firms' behaviour is freely accessible (costless)…
- … but, in some circumstances, it could be individually convenient to ignore such information
- Two int
  - ○ the fi
    such
  - ○ the value of certain information can be negative.

The information is not relevant per se, but rather for the way in which it affects the best reply actions

Literature discussing the value of information in games, where players rationally prefer to ignore some potentially relevant pieces of information (Bassan, Scarsini, Zamir, 1997; Bassan, Gossner, Scarsini, Zamir, 2003; Kamien, Tauman, Zamir, 1990).

Structure of the presentation

- Basics: The two-firm model
- The *N*-firm model:
  - Unilateral deviation from the “Fully Oligopolistic framework” (O) , and from the “Fully Monopolistically Competitive framework” (MC)
  - Stability of a partition of firms’ population between O and MC
  - Simple numerical simulations
  - Theoretical conclusions
- Comments and implications

The two-firm model

- Two single-product firms (*i*, *j*) operate in a market for differentiated products,
- Individual inverse market demand

(Bowley 1924, Spence, 1976, Singh-Vives, 1984, …)

$$p_i = a - \gamma q_i - \sigma q_j$$

$a>0,\ \gamma>0,\ 0\leq\sigma\leq\gamma$ *(subst. goods)*

Limiting cases: $\sigma=0$ (max differentiation)

$\sigma=\gamma$ (min differentiation = homog. olig)

- Differentiated oligopoly; Firms set output levels to maximise individual profits (Cournot).

The two-firm model

- The market demand is rewritten as:

$$p_i = a - \gamma q_i - \sigma q_j + \sigma q_i - \sigma q_i$$

$$p_i = a - (\gamma + \sigma) q_i - \sigma q_j$$

$$p_i = a - \beta q_i - \sigma (q_i + q_j)$$

$$p_h = a - \beta q_h - \sigma Q \quad \text{with} \quad \gamma \equiv \beta + \sigma \ , \ (h=i,j)$$

- Condition $0 \leq \sigma \leq \gamma$ corresponds to $0 \leq \sigma \leq (\beta+\sigma)$ ➔ $\beta \geq 0$
- σ: How much sensitive p_h is to the quantity of the opponent
- If $\sigma = 0$, maximum product differentiation
- If $\beta = 0$, identical products (homogenous oligopoly)

The two-firm model

- The market demand is r

$$p_i = a - \gamma q_i - \sigma q_j + \sigma q_i -$$

$$p_i = a - (\gamma + \sigma) q_i - \sigma q_j$$

$$p_i = a - \beta q_i - \sigma (q_i + q_j)$$

$$p_h = a - \beta q_h - \overline{\underline{\sigma Q}} \quad \text{with} \quad \gamma \equiv \beta + \sigma \ , \ (h = i,j)$$

The core-issue:
How has Q to be considered?
Exogeneous or endogeneous?

- Condition $0 \leq \sigma \leq \gamma$ corresponds to $0 \leq \sigma \leq (\beta + \sigma)$ ➔ $\beta \geq 0$
- σ: How much sensitive p_h is to the quantity of the opponent
- If $\sigma = 0$, maximum product differentiation
- If $\beta = 0$, identical products (homogenous oligopoly)

The two-firm model

- Cost function: $c_h = cq_h + bq_h^2$ (with $c \geq 0$, $b \geq 0$)
- Profit: $\pi_h = (p_h - c)q_h - bq_h^2$.

[In the new version:

$$\alpha \equiv ((a-c)/(\beta+b))\ , \quad \eta \equiv (\sigma/(\beta+b))\ , \quad \ell \equiv \beta+b$$

$$\pi_h = \ell(\alpha - q_h - \eta Q)q_h$$

key parameter : η. This measures the impact of aggregate output Q on the firm's profit margin π_h/q_h

η measures the dependence of the firm's profit on the industry aggregate. In the limit case η=0 the firm's profit is independent from aggregate output.

As η grows, the firm's profit increasingly depend on aggregate output.

Hence, we call η the `aggregate dependence' parameter.

The two-firm model

- Cost function: $c_h = cq_h + bq_h^2$

  (with $c \geq 0$, $b \geq 0$)
- Profit: $\pi_h = (p_h - c)q_h - bq_h^2$.

The Cournot oligopoly equilibrium:

FOCs:

$$\frac{\partial \pi_i}{\partial q_i} = a - 2(b + \beta + \sigma)q_i - \sigma q_j - c = 0$$

$$\frac{\partial \pi_j}{\partial q_j} = a - 2(b + \beta + \sigma)q_j - \sigma q_i - c = 0$$

The two-firm model

The Cournot oligopoly equilibrium:

$$q^{ss} = \frac{a-c}{2(b+\beta)+3\sigma}$$

$$\pi^{ss} = \frac{(a-c)^2(b+\beta+\sigma)}{[2(b+\beta)+3\sigma]^2}$$

- Superscript *ss*: smart-smart
- (We denote as '***smart***' the firm taking into account the interdependence, and '***myopic***' the firms that does not).
- Smart = oligopolistic firms
- Myopic = Monopolistically competitive firms

The two-firm model

If both firms are unaware of their roles when setting the optimal quantities :

- individual market demand perceives as: $p_h = a - \beta q_h - \sigma \overline{Q}$
- FOC: $\frac{\partial \pi_i}{\partial q_i} = a - 2(b + \beta)q_i - \sigma \overline{Q} - c = 0.$
- We plug $Q = q_i + q_j$ only after the optimal choices are made
- Individual firm's behaviour: $a - 2(b + \beta)q_i - \sigma(q_i + q_j) - c = 0$
- Production levels: $q^{mm} = \frac{a - c}{2(b + \beta + \sigma)}.$
- Profit levels: $\pi^{mm} = \frac{(a - c)^2 (b + \beta)}{4(b + \beta + \sigma)^2}.$

The two-firm model

The «mixed» setting:

- Firm *i* behaves as an oligopolist:

(with FOC $$\frac{\partial \pi_i}{\partial q_i} = a - 2(b+\beta+\sigma)q_i - \sigma q_j - c = 0$$

- Firm *j* behaves as a MC subject:

(with FOC $$\frac{\partial \pi_i}{\partial q_i} = a - 2(b+\beta)q_i - \sigma Q - c = 0.$$

- Production levels are:

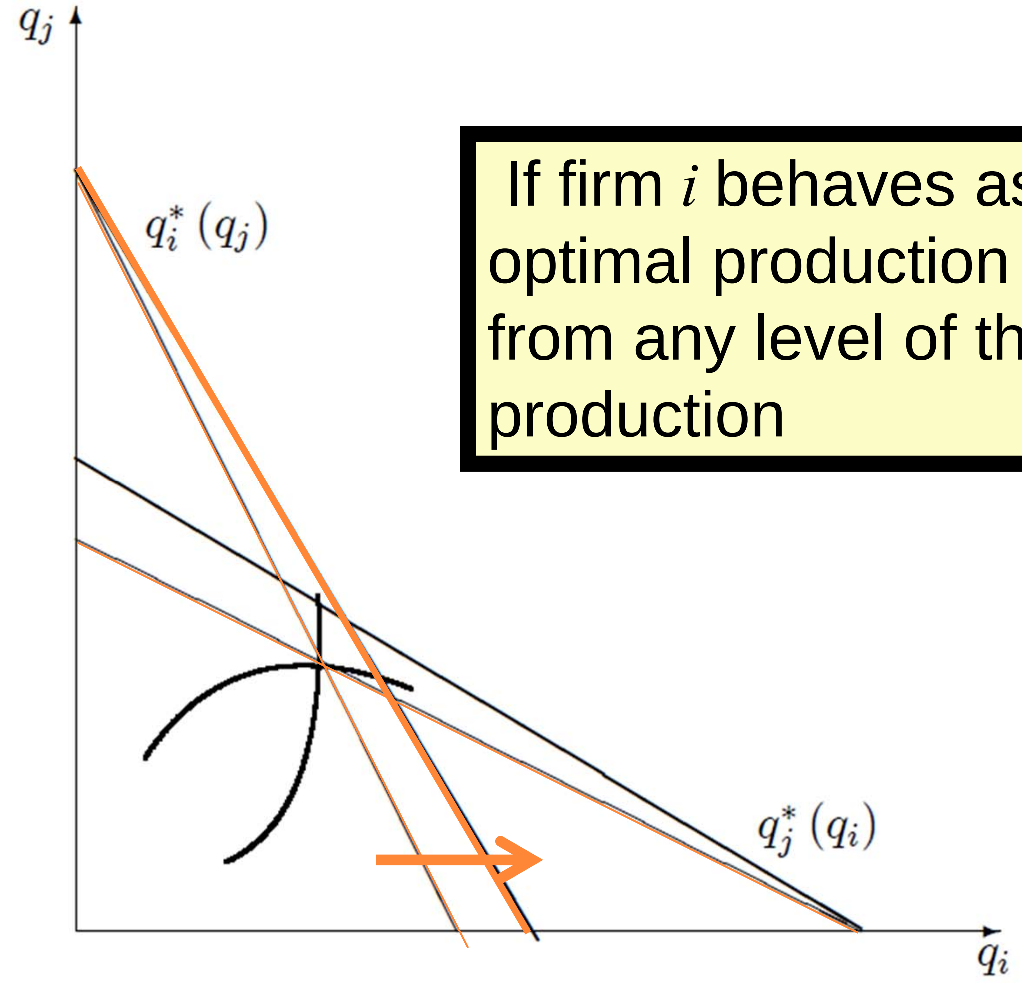
$$q_i^{sm} = \frac{2(a-c)(b+\beta)}{4(b+\beta)^2 + \sigma[\sigma + 6(\beta+\sigma)]}\,;\, q_j^{ms} = \frac{(a-c)[2(b+\beta)+\sigma]}{4(b+\beta)^2 + \sigma[\sigma + 6(\beta+\sigma)]}$$

- Profit levels are:

$$\pi_i^{sm} = \frac{4(a-c)^2(b+\beta)^2(b+\beta+\sigma)}{\left[4(b+\beta)^2 + \sigma(\sigma + 6(\beta+\sigma))\right]^2}\,;\, \pi_j^{ms} = \frac{(a-c)^2(b+\beta)[2(b+\beta)+\sigma]^2}{\left[4(b+\beta)^2 + \sigma(\sigma + 6(\beta+\sigma))\right]^2}.$$

The two-firm model

- In the mixed setting: $q_i^{sm} < q_j^{ms}$ and $\pi_i^{sm} < \pi_j^{ms}$
- (the MC –“myopic”– firm is bigger and richer than the O –“smart”– firm)
- The reason is the same as in delegation model *à la* Vickers.
- … It is *as if* the “reaction function” of the myopic firm moves outwards



If firm i behaves as a MC firm, its optimal production increases, from any level of the opponent's production

The two-firm model

- Now let us imagine that the behaviour (whether or not to consider the fact that $Q=q_i+q_j$) pertains to a «pre-market» stage.
- Each firm can choose whether to behave as an oligopolist (to be«smart») or in a Monopolistically Competitive way (to be or «myopic»).
- A simultaneous game under complete and imperfect info.

$$\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{2} \\
 & & s & m \\
\hline
1 & s & \pi^{ss}, \pi^{ss} & \pi^{sm}, \pi^{ms} \\
\hline
 & m & \pi^{ms}, \pi^{sm} & \pi^{mm}, \pi^{mm} \\
\hline
\end{array}$$

Matrix 0: The first stage

The two-firm model

- The comparison of π^{ss} , π^{mm} , π^{sm} , π^{ms} is easy:
- $\pi^{ss} > \pi^{mm}$; $\pi^{ss} > \pi^{ms}$; $\pi^{ms} > \pi^{sm}$; $\pi^{mm} >^{?}< \pi^{sm}$
- Two kinds of game are possible:
- a single and Pareto-efficient pure-strategy equilibrium at (s,s), generated by the intersection of dominant strategies [when parameter σ is rather small ($\sigma < 6b/5$) and parameter β is larger than a threshold level; or, equivalently, $\eta<2(1+2^{1/2})$].
- a coordination game, with two pure-strategy Nash equilibria in (s,s) and (m,m) [whenever σ is large ($\sigma >$ 6b/5), or in the case $\sigma \leq 6b/5$ joint with β smaller than a threshold level; equivalently $\eta>2(1+2^{1/2})$].

The two-firm model

- To be clearer, two numerical cases.

  $(a\text{-}c)^2=100,\ b=1,\ \beta=1;$ then (A): $\sigma=1$ (B): $\sigma=2$.

Case (A)

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{2} \\ & & s & m \\ \hline 1 & s & \underline{42.8};\ \underline{42.8} & \underline{5.7};\ 9.9 \\ \hline & m & 9.9;\ \underline{5.7} & 5.5;\ 5.5 \\ \hline \end{array}$$

Case (B)

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{2} \\ & & s & m \\ \hline 1 & s & \underline{100};\ \underline{100} & 3.3;\ 3.7 \\ \hline & m & 3.7;\ 3.3 & \underline{12.5};\ \underline{12.5} \\ \hline \end{array}$$

The two-firm model

Under parameter constallation corresponding to case (B), the «fully myopic» setting is an equilibrium:

- Provided that the opponent chooses to behave as a MC firm, it is individually optimal to adopt the MC behaviour.
- However MC-MC is Pareto-inefficient
- Even O-O is an equilibrium (Pareto-efficient!)
- Of course, in this case (B), also an equilibrium in mixed strategy exists (which is Pareto inefficient as compared to O-O).

The two-firm model – Digression 1

- (A digression in the paper shows that our two-firm model analytically resembles the delegation model of Miller and Pazgal (2001), where two owners of two oligopolistic firms (behaving *à la* Cournot in the market phase) have to decide (in a pre-market phase) whether or not to delegate managers the choice about production, and they design the contract for the manager in order to maximise the firms' individual profit – where the delegation contract is based on comparative profit performances.)

The two-firm model – Digression 2

- A short additional digression is made, concerning other mechanisms that lead firms to disregard information:
- Specifically, ‘***divisionalization***’ (Baye et al., 1996; Ziss, 1998)
  - a large (multiproduct) firm may delegate the choices concerning any single product to an independent manager unaware of interdependencies among products.
  - In Kokovin et al (2014): a big (oligopolistic) firm competes with a fringe of small monopolistically competitive rivals; depending on the market demand configuration, the big firm may find it convenient to be broken down into horizontal profit-maximizing divisions that disregard their interdependencies and behave like MC units.

The N-firm model

- A more general case
- The same market (i.e., under the same assumptions concerning the demand system and technology), is served by *N* single-product firms.
- The demand side – individual demand:

$$p_i = a - \gamma q_i - \sigma(Q_{-i})$$

- Rewritten as:

$$p_i = a - \beta q_i - \sigma(Q)$$

- with

$$\gamma \equiv \beta + \sigma$$

The N-firm model

- Two related perspectives are considered.

- A) consider a single firm that chooses whether to be smart or myopic given a homogeneous choice by the remaining (*N*-1) firms.

  ➔ The existence (or lack) of an individual profit incentive to deviate from either fully symmetric outcome in which the entire industry is alternatively smart or myopic.

- B) consider a generic composition of the industry, assuming that *K* firms are myopic and the remaining (*N*-*K*) are smart.

  ➔ The existence (or the lack) of a stable partition of the population of firms into a smart group and a myopic one.

The N-firm model
- The «fully smart» (oligopoly) equilibrium -

- Demand function system: $p_i = a - \beta q_i - \sigma\left(q_i + \sum_{j \neq i} q_j\right).$
- The FOCs for firm *i*: $\frac{\partial \pi_i}{\partial q_i} = a - 2(b + \beta + \sigma)q_i - \sigma \sum_{j \neq i} q_j - c = 0$
- Assuming symmetry: $q_i = q_j = q$; $\sum_{j \neq i} q_j = (n-1)q$
- The equilibrium output: $q^{ss}(n) = \frac{a-c}{2(b+\beta) + \sigma(n+1)}$
- The individual eqil. profit: $\pi^{ss}(n) = \frac{(a-c)^2(b+\beta+\sigma)}{[2(b+\beta)+\sigma(n+1)]^2}.$

The N-firm model

- The «fully smart» (oligopoly) equilibrium -

- Consider now the case that a single firm *i* evaluates the possibility of becoming myopic (given that all remaining (*N*-1) firms continue to be smart.

- ➔ firm *i* has $\frac{\partial \pi_i}{\partial q_i} = a - 2(b+\beta)q_i - \sigma Q - c = 0$ as its FOC,
- ➔ the FOC for each of its (*N*-1) smart rival is :

$$\frac{\partial \pi_j}{\partial q_j} = a - 2(b+\beta+\sigma)q_j - \sigma Q_{-j} - c = 0$$

with $Q_{-j} = \sum_{\ell \neq j} q_\ell$.

- The above system is solved by a vector of outputs composed by a single quantity q^{ms} and *N*-1 quantities q^{sm}

The N-firm model
- The «fully smart» (oligopoly) equilibrium -

- Solution:

$$q^{ms}(1,n-1)=\frac{(a-c)[2(b+\beta)+\sigma]}{4(b^2+\beta^2)+\sigma[\sigma+2b(n+1)]+2\beta[4b+\sigma(n+1)]}$$

$$q^{sm}(n-1,1)=\frac{2(a-c)(b+\beta)}{4(b^2+\beta^2)+\sigma[\sigma+2b(n+1)]+2\beta[4b+\sigma(n+1)]}$$

It is : $q^{ms}(1,n\text{-}1)>q^{sm}(n\text{-}1,1)$

If all rivals are smart, being individually myopic allows a single firm to expand output, irrespective of the overall number of firms in the industry.

- Correspondingly, profit levels are: …

The N-firm model

- The «fully smart» (oligopoly) equilibrium -

- Profits:

$$\pi^{ms}(1,n-1)=\frac{(a-c)^2[2(b+\beta)+\sigma]^2(b+\beta)}{[4(b^2+\beta^2)+\sigma(\sigma+2(b+\beta)(n+1))]^2}$$

$$\pi^{sm}(n-1,1)=\frac{4(a-c)^2(b+\beta)^2(b+\beta+\sigma)}{[4(b^2+\beta^2)+\sigma(\sigma+2(b+\beta)(n+1))]^2}.$$

- *The unilateral deviation from smart to myopic is convenient iff* $\pi^{ms}(1,n\text{-}1) > \pi^{ss}(n)$ *, i.e., iff :*

$$(b+\beta)[\sigma^2(n(n-2)-4)-4(b+\beta)(b+\beta+\sigma)]-\sigma^3 > 0.$$

- (We will study it, but a necessary condition to meet is $n \geq 4$)

The N-firm model

- The «fully smart» (oligopoly) equilibrium -

- Profitability of unilateral deviation from the «fully smart» allocation, i.e., $(b+\beta)[\sigma^2(n(n-2)-4)-4(b+\beta)(b+\beta+\sigma)]-\sigma^3 > 0.$
- *Met in the region where* $(b+\beta+\sigma)[2(b+\beta)+\sigma] > 9(b+\beta)\sigma^2$

  and $n>n_+$*, (where* $n_\pm = \dfrac{\sigma(b+\beta) \pm [2(b+\beta)+\sigma]\sqrt{(b+\beta)(b+\beta+\sigma)}}{\sigma(b+\beta)}$ *), that is,*
- *Met in the region where* $n>1+[1+(2/\eta)](\eta+1)^{1/2}$.

That is to say, *for a firm to find it convenient to be myopic in front of a population of smart rivals, the industry must be sufficiently fragmented (with the threshold, related to demand parameters).*

The N-firm model

- The «fully smart» (oligopoly) equilibrium -

CLEAR & INTUITIVE CONCLUSION
(from a didactical point of view):

If a market starts from a situation in which an oligopoly exists, is it possible that a firm changes its behaviour and starts behaving as a monopolistic competitive unit?

No, it isn’t, if the number of firms is limited

Yes, it could be possible if the number of related firms is large

(the NUMBER of firms matters!)

The N-firm model

- The «fully myopic» (MC) equilibrium -

- Demand function system: $p_i = a - \beta q_i - \sigma \overline{Q}.$
- The FOCs for firm i: $\frac{\partial \pi_i}{\partial q_i} = a - 2(b+\beta)q_i - \sigma Q - c = 0.$
- Assuming symmetry: q_i=q_j=q ; $\sum_{j \neq i} q_j = (n-1)q$
- The equilibrium output: $q^{mm}(n) = \frac{a-c}{2(b+\beta)+n\sigma}$
- The individual equilibrium profit: $\pi^{mm}(n) = \frac{(a-c)^2(b+\beta)}{[2(b+\beta)+n\sigma]^2}.$

The N-firm model
- The «fully myopic» (MC) equilibrium -

- Consider now the case that a single firm *i* evaluates the possibility of becoming smart (given that all remaining (*N*-1) firms continue to be myopic.
- ➔ firm *i* has $\frac{\partial \pi_i}{\partial q_i} = a - 2(b + \beta + \sigma)q_i - \sigma Q_{-i} - c = 0$ as its FOC, while the FOC for each of its (*N*-1) myopic rival is :

$$\frac{\partial \pi_j}{\partial q_j} = a - 2(b + \beta)q_j - \sigma Q - c = 0$$

- Imposing symmetry (after having computed the FOC) , the equilibrium output levels obtain : …

The N-firm model

- The «fully myopic» (MC) equilibrium -

- Ouput levels:

$$q^{sm}(1,n-1) = \frac{2(a-c)(b+\beta)}{4(b^2+\beta^2)+\sigma[\sigma+2b(n+1)]+2\beta[4b+\sigma(n+1)]}$$

$$q^{ms}(n-1,1) = \frac{(a-c)[2(b+\beta)+\sigma]}{4(b^2+\beta^2)+\sigma[\sigma+2b(n+1)]+2\beta[4b+\sigma(n+1)]}$$

- with $q^{sm}(1,n\text{-}1)<q^{ms}(n\text{-}1,1)$, (but $q^{sm}(1,n\text{-}1) >^{?}< q^{mm}$).
- Corresponding profit levels:

$$\pi^{sm}(1,n-1) = \frac{4(a-c)^2(b+\beta)^2(b+\beta+\sigma)}{[2(b+\beta)(2(b+\beta)+(n+1)\sigma)+(n-1)\sigma^2]^2}$$

$$\pi^{ms}(n-1,1) = \frac{(a-c)^2(b+\beta)[2(b+\beta)+\sigma]^2}{[2(b+\beta)(2(b+\beta)+(n+1)\sigma)+(n-1)\sigma^2]^2}$$

The N-firm model
- The «fully myopic» (MC) equilibrium -

- The unilateral deviation from the fully myopic allocation is profitable iff : $\pi^{sm}(1,n\text{-}1) - \pi^{mm}(n) > 0$.

- It is easy to check:

$$\pi^{sm}(1,n\text{-}1) - \pi^{mm}(\mathrm{n}) \propto 4(b+\beta)[b+\beta+\sigma]\text{-}(n\text{-}1)^2\sigma^2 > 0$$

- This disequality is satisfied for: $\sigma \in \left(0, \frac{2\left(1+\sqrt{2+n(n-2)}\right)(b+\beta)}{(n-1)^2}\right)$

- That is, $(n-1) < \frac{2}{\sigma}\sqrt{(b+\beta)(b+\beta+\sigma)}$
- Or, equivalently, $n<1+(2/\eta)(1+\eta)^{1/2}$.

The N-firm model

- The «fully myopic» (MC) equilibrium -

Comments:

- The fully myopic outcome is always Pareto-inefficient with respect to the fully smart outcome

- What about the individual incentive to deviate from the fully myopic outcome?
- (When the fully myopic outcome is an equilibrium?, that is, When does it happen that no individual incentive exists to deviate from it?)
- Two ways for telling the result

The N-firm model

- The «fully myopic» (MC) equilibrium -

- (1) NOT VERY INTUITIVE WAY OF TELLING THE RESULT:
- Profitable unilateral deviation if $\sigma \in \left(0, \frac{2\left(1+\sqrt{2+n(n-2)}\right)(b+\beta)}{(n-1)^2}\right)$
- If the impact of aggregate industry output on individual performance - measured by parameter *σ* - is not too relevant (i.e., high degree of product differentiation), the unilateral deviation from the fully myopic outcome is profitable.
- On the contrary, if parameter *σ* is large (entailing a low degree of product differentiation) the unilateral deviation from the fully myopic equilibrium is not profitable.
- (To be honest, …)

The N-firm model

- The «fully myopic» (MC) equilibrium -

- (2) VERY INTUITIVE WAY OF TELLING THE RESULT:
- Profitable unilateral deviation if $(n-1) < \frac{2}{\sigma}\sqrt{(b+\beta)(b+\beta+\sigma)}$
- Only if the number of the firms serving the market is limited, it can be individually convenient to behave as an oligopolistic firm, provided that the status quo is the fully myopic setting.

The N-firm model

- The «fully myopic» (MC) equilibrium -

- (2) VERY INTUITIVE WAY OF TELLING THE RESULT:
- Profitable unilateral deviation if $(n-1) < \frac{2}{\sigma}\sqrt{(b+\beta)(b+\beta+\sigma)}$
- Only if the number of the firms serving the market is limited, it can be individually convenient to behave as an oligopolistic firm, provided that the status quo is the fully myopic setting.

CLEAR & INTUITIVE CONCLUSION
(from a didactical point of view):
- the NUMBER of firms matters!
- Truly, the number of firms interacts with the degree of product substitutability.

The N-firm model

- Is there a stable partition? -

- (This Section is largely indebted to the coalition theory)
- The industry consists of K=1,2,3,…k myopic firms and
  N-K=k+1,k+2,k+3,…N smart ones.
- To characterise the game, look at two of these firms,
  a smart one (i) and a myopic one (j).
- They face the demand functions: $p_i = a - \beta q_i - \sigma\left(q_i + \sum_{\ell \neq i} q_\ell\right)$

$$p_j = a - \beta q_j - \sigma\overline{Q}$$

- FOCs are respectively: $\dfrac{\partial \pi_i}{\partial q_i} = a - 2(b+s)q_i - s(q_j + Q_{K-j} + Q_{N-K-i}) - c = 0$

$$\frac{\partial \pi_j}{\partial q_j} = a - 2(b+\beta)q_j - sQ - c = 0$$

$Q_{K\text{-}j}$: the collective output of the myopic group (except j)

$Q_{N\text{-}K\text{-}i}$: the collective output of the smart group (except i)

The N-firm model

- Is there a stable partition? -

- Solving the system of FOCs –under the symmetry assumptions: $Q_{K\text{-}j} = (k\text{-}1)q^m$, $Q_{N\text{-}K\text{-}i} = (N\text{-}k\text{-}1)q^s$:

$$q^{sm}(n-k,k) = \frac{2(a-c)(b+\beta)}{4(b^2+\beta^2)+\sigma[\sigma k+2\beta(n+1)]+2b[4\beta+(n+1)\sigma]}$$

$$q^{ms}(k,n-k) = \frac{(a-c)[2(b+\beta)+\sigma]}{4(b^2+\beta^2)+\sigma[\sigma k+2\beta(n+1)]+2b[4\beta+(n+1)\sigma]}$$

- Corresponding profit levels are:

$$\pi^{sm}(n-k,k) = \frac{4(a-c)^2(b+\beta)^2(b+\beta+\sigma)}{\left[4(b+\beta)^2+\sigma(\sigma k+2(b+\beta)(n+1))\right]^2}$$

$$\pi^{ms}(k,n-k) = \frac{(a-c)^2(b+\beta)[2(b+\beta)+\sigma]^2}{\left[4(b+\beta)^2+\sigma(\sigma k+2(b+\beta)(n+1))\right]^2}.$$

The N-firm model

- Is there a stable partition? -

- In the mixed setting, irrespective of the given number and composition of the population of firms:

  $q^{ms}(k,n\text{-}k) > q^{sm}(n\text{-}k,k)$ and

  $\pi^{ms}(k,n\text{-}k) > \pi^{sm}(n\text{-}k,k)$.
- (production and profit of a myopic firm are larger)

- … But to evaluate the individual incentives to deviate, one has to consider that …
- … If a firm changes its behaviour, the partition does change

The N-firm model

- Is there a stable partition? -

For a partition to be stable, two conditions must hold:

- A] - it must not be individually profitable for a myopic firm in *K* to abandon this behaviour and join the set of smart firms (in which case the number of myopic firms would decrease to *k*-1 while that of smart ones would increase to *N*-*k*+1);
- B] - it must not be individually profitable for a smart firm in *N*-*K* to quit this group to become a myopic one (in which case the number of myopic firms would increase to *k*+1 while that of smart ones would decrease to *N*-*k*-1).

The N-firm model
- Is there a stable partition? -

- A] means: $\pi^{ms}(k, n-k) \geq \pi^{sm}(n-k+1, k-1) \Leftrightarrow$

$$\frac{[2(b+\beta)+\sigma]^2}{\left[4(b+\beta)^2+\sigma(\sigma k+2(b+\beta)(n+1))\right]^2} \geq \frac{4(b+\beta)(b+\beta+\sigma)}{\left[4(b+\beta)^2+\sigma(\sigma(k-1)+2(b+\beta)(n+1))\right]^2}$$

- B] means: $\pi^{sm}(n-k, k) \geq \pi^{ms}(k+1, n-k-1) \Leftrightarrow$

$$\frac{4(b+\beta)(b+\beta+\sigma)}{\left[4(b+\beta)^2+\sigma(\sigma k+2(b+\beta)(n+1))\right]^2} \geq \frac{[2(b+\beta)+\sigma]^2}{\left[4(b+\beta)^2+\sigma(\sigma(k+1)+2(b+\beta)(n+1))\right]^2}$$

The N-firm model
- Is there a stable partition? -

- A] means: $\pi^{ms}(k, n-k) \geq \pi^{sm}(n-k+1, k-1) \Leftrightarrow$

$$\frac{[2(b+\beta)+\sigma]^2}{\left[4(b+\beta)^2+\sigma(\sigma k+2(b+\beta)(n+1))\right]^2} \geq \frac{4(b+\beta)(b+\beta+\sigma)}{\left[4(b+\beta)^2+\sigma(\sigma(k-1)+2(b+\beta)(n+1))\right]^2}$$

- B] means: $\pi^{sm}(n-k, k) \geq \pi^{ms}(k+1, n-k-1) \Leftrightarrow$

$$\frac{4(b+\beta)(b+\beta+\sigma)}{\left[4(b+\beta)^2+\sigma(\sigma k+2(b+\beta)(n+1))\right]^2} \geq \frac{[2(b+\beta)+\sigma]^2}{\left[4(b+\beta)^2+\sigma(\sigma(k+1)+2(b+\beta)(n+1))\right]^2}$$

A long and tedious (but simple) proof can be provided that [A] and [B] cannot be satisfied simultaneously!

The N-firm model

- Is there a stable partition? -

- More formally:

The intervals of *k* wherein, respectively,

$$\pi^{ms}(k, n-k) \geq \pi^{sm}(n-k+1, k-1);$$

$$\pi^{sm}(n-k, k) \geq \pi^{ms}(k+1, n-k-1),$$

are disjoint for all admissible values of parameters $\{b,n,\beta,\sigma\}$. Consequently, there exists no stable partition of the population of firms between the smart and the myopic group.

- We already proved that :

The fully-myopic allocation, and the fully-smart allocation can be stable equilibrium or not, depending on parameter configuration.

- We have a final Proposition to prove:

The N-firm model

- Is there a stable partition? -

- For any given parameter configuration it is impossible that the fully-smart allocation and the fully-myopic allocation are simultaneously unstable.

  ➔ For any given parameter configuration, the instability of the fully-smart allocation implies that the fully myopic allocation is an equilibrium, and the instability of the fully-myopic allocation implies that the fully-smart allocation is an equilibrium.

- (The proof of this Proposition is quick and simple)

The N-firm model

- Is there a stable partition? -

Summing, up:

- No stable partition of firms does exist in which heterogenous behaviours co-exist;
- The simultaneous instability of the full myopic and the full smart allocation is impossible.

➔ *These properties prevent the perpetual mobility of firms across smart and myopic groups to be observable.*

- The fully-smart or the fully-myopic allocation (or both) are stable.
- The fully myopic allocation is Pareto-inefficient w.r.t. the fully smart one.

The N-firm model

- Is there a stable partition? -

Corollary (and conclusion):

In the game in which firms can choose whether to consider or not the strategic interdependence, i.e., they can choose whether to behave as smart or myopic firms, tree cases are possible:

(1) the unique equilibrium is the situation in which all firms choose to take into account the strategic interdependence ("*fully smart allocation*");

(2) the unique equilibrium is the situation where all firms choose to disregard the strategic interdependence ("*fully myopic allocation*");

(3) both the "fully smart" and the "fully myopic" allocations are equilibria.

In all cases, the fully myopic allocation is Pareto inefficient w.r.t. the fully smart allocation, irrespective of the equilibrium situation.

The N-firm model

- Is there a stable partition? -

Numerical simulations (b=0; $(a\text{-}c)^2$=100)

	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$n=4,$ $A)\ \sigma=.005$ $\beta=.1$	$(4)\pi^s=207.4$.	$(3)\pi^s=207.1$ $(1)\pi^m=207.3$	$(2)\pi^s=206.9$ $(2)\pi^m=207.0$	$(1)\pi^s=206.7$ $(3)\pi^m=206.8$	. $(4)\pi^m=206.6$	
$n=4,$ $B)\ \sigma=.01$ $\beta=.1$	$(4)\pi^s=40.8$.	$(3)\pi^s=35.5$ $(1)\pi^m=40.0$	$(2)\pi^s=31.2$ $(2)\pi^m=35.1$	$(1)\pi^s=27.6$ $(3)\pi^m=31.1$	. $(4)\pi^m=27.7$	
$n=5,$ $C)\ \sigma=.001$ $\beta=.0001$	$(5)\pi^s=2861$.	$(4)\pi^s=876$ $(1)\pi^m=2869$	$(3)\pi^s=419$ $(2)\pi^m=1371$	$(2)\pi^s=244$ $(3)\pi^m=800$	$(1)\pi^s=160$ $(4)\pi^m=524$	. $(5)\pi^m=396$

T

- Is th

First, notice that:

In any mixed setting, for any given partition,

$\pi^{ms}(k,n-k)>\pi^{sm}(n-k,k)$

Numerical s

	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$A)$ $n=4,$ $\sigma=.005$ $\beta=.1$	$(4)\pi^s=207.4$.	$(3)\pi^s=207.1$ $(1)\pi^m=207.3$	$(2)\pi^s=206.9$ $(2)\pi^m=207.0$	$(1)\pi^s=206.7$ $(3)\pi^m=206.8$	. $(4)\pi^m=206.6$	
$B)$ $n=4,$ $\sigma=.01$ $\beta=.1$	$(4)\pi^s=40.8$.	$(3)\pi^s=35.5$ $(1)\pi^m=40.0$	$(2)\pi^s=31.2$ $(2)\pi^m=35.1$	$(1)\pi^s=27.6$ $(3)\pi^m=31.1$	. $(4)\pi^m=27.7$	
$C)$ $n=5,$ $\sigma=.001$ $\beta=.0001$	$(5)\pi^s=2861$.	$(4)\pi^s=876$ $(1)\pi^m=2869$	$(3)\pi^s=419$ $(2)\pi^m=1371$	$(2)\pi^s=244$ $(3)\pi^m=800$	$(1)\pi^s=160$ $(4)\pi^m=524$	. $(5)\pi^m=396$

T

- Is th

Second, notice that:

In any setting, the fully myopic allocation is Pareto-inefficient

Numerical s

	$k = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$n = 4,$ $A)\ \sigma = .005$ $\beta = .1$	$(4)\pi^s = 207.4$.	$(3)\pi^s = 207.1$ $(1)\pi^m = 207.3$	$(2)\pi^s = 206.9$ $(2)\pi^m = 207.0$	$(1)\pi^s = 206.7$ $(3)\pi^m = 206.8$	. $(4)\pi^m = 206.6$	
$n = 4,$ $B)\ \sigma = .01$ $\beta = .1$	$(4)\pi^s = 40.8$.	$(3)\pi^s = 35.5$ $(1)\pi^m = 40.0$	$(2)\pi^s = 31.2$ $(2)\pi^m = 35.1$	$(1)\pi^s = 27.6$ $(3)\pi^m = 31.1$	. $(4)\pi^m = 27.7$	
$n = 5,$ $C)\ \sigma = .001$ $\beta = .0001$	$(5)\pi^s = 2861$.	$(4)\pi^s = 876$ $(1)\pi^m = 2869$	$(3)\pi^s = 419$ $(2)\pi^m = 1371$	$(2)\pi^s = 244$ $(3)\pi^m = 800$	$(1)\pi^s = 160$ $(4)\pi^m = 524$	. $(5)\pi^m = 396$

T

- Is th

Evaluate the **individual incentive** to change behaviour in case (A)

Numerical simulations (b=0; $(a-c)^2$=100)

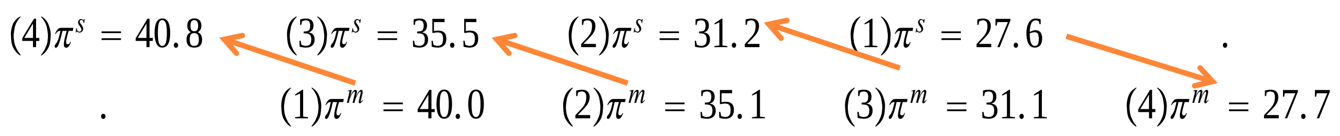
	$k = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$A)$ $n = 4,$ $\sigma =.005$ $\beta =.1$	$(4)\pi^s = 207.4$.	$(3)\pi^s = 207.1$ $(1)\pi^m = 207.3$	$(2)\pi^s = 206.9$ $(2)\pi^m = 207.0$	$(1)\pi^s = 206.7$ $(3)\pi^m = 206.8$	. $(4)\pi^m = 206.6$	
$B)$ $n = 4,$ $\sigma =.01$ $\beta =.1$	$(4)\pi^s = 40.8$.	$(3)\pi^s = 35.5$ $(1)\pi^m = 40.0$	$(2)\pi^s = 31.2$ $(2)\pi^m = 35.1$	$(1)\pi^s = 27.6$ $(3)\pi^m = 31.1$	. $(4)\pi^m = 27.7$	
$C)$ $n = 5,$ $\sigma =.001$ $\beta =.0001$	$(5)\pi^s = 2861$.	$(4)\pi^s = 876$ $(1)\pi^m = 2869$	$(3)\pi^s = 419$ $(2)\pi^m = 1371$	$(2)\pi^s = 244$ $(3)\pi^m = 800$	$(1)\pi^s = 160$ $(4)\pi^m = 524$	. $(5)\pi^m = 396$

T

- Is th

Evaluate the individual incentive to change behaviour in case (B)

Numerical simulations (b=0; $(a\text{-}c)^2$=100)



	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$n=4,$ $A)\ \sigma=.005$ $\beta=.1$	$(4)\pi^s=207.4$.	$(3)\pi^s=207.1$ $(1)\pi^m=207.3$	$(2)\pi^s=206.9$ $(2)\pi^m=207.0$	$(1)\pi^s=206.7$ $(3)\pi^m=206.8$	. $(4)\pi^m=206.6$	
$n=4,$ $B)\ \sigma=.01$ $\beta=.1$	$(4)\pi^s=40.8$.	$(3)\pi^s=35.5$ $(1)\pi^m=40.0$	$(2)\pi^s=31.2$ $(2)\pi^m=35.1$	$(1)\pi^s=27.6$ $(3)\pi^m=31.1$	. $(4)\pi^m=27.7$	
$n=5,$ $C)\ \sigma=.001$ $\beta=.0001$	$(5)\pi^s=2861$.	$(4)\pi^s=876$ $(1)\pi^m=2869$	$(3)\pi^s=419$ $(2)\pi^m=1371$	$(2)\pi^s=244$ $(3)\pi^m=800$	$(1)\pi^s=160$ $(4)\pi^m=524$	. $(5)\pi^m=396$

T

- Is th

Evaluate the **individual incentive** to change behaviour in case (C)

Numerical simulations (b=0; $(a\text{-}c)^2$=100)

	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$n=4,$ $A)\ \sigma=.005$ $\beta=.1$	$(4)\pi^s=207.4$.	$(3)\pi^s=207.1$ $(1)\pi^m=207.3$	$(2)\pi^s=206.9$ $(2)\pi^m=207.0$	$(1)\pi^s=206.7$ $(3)\pi^m=206.8$	. $(4)\pi^m=206.6$	
$n=4,$ $B)\ \sigma=.01$ $\beta=.1$	$(4)\pi^s=40.8$.	$(3)\pi^s=35.5$ $(1)\pi^m=40.0$	$(2)\pi^s=31.2$ $(2)\pi^m=35.1$	$(1)\pi^s=27.6$ $(3)\pi^m=31.1$	. $(4)\pi^m=27.7$	
$n=5,$ $C)\ \sigma=.001$ $\beta=.0001$	$(5)\pi^s=2861$.	$(4)\pi^s=876$ $(1)\pi^m=2869$	$(3)\pi^s=419$ $(2)\pi^m=1371$	$(2)\pi^s=244$ $(3)\pi^m=800$	$(1)\pi^s=160$ $(4)\pi^m=524$	. $(5)\pi^m=396$

T

- Is th

Evaluate the **individual incentive** to change behaviour in case (C)

Case C is very interesting:

The fully myopic allocation is an equilibrium, but it is inefficient;

The fully smart allocation is efficient, but it is not an equilibrium;

(it resembles the prisoner's dilemma story!)

Numerical s

	$k = 0$	k				
$A)$ $n = 4,$ $\sigma = .005$ $\beta = .1$	$(4)\pi^s = 207.4$.	$(3)\pi^s$ $(1)\pi^m$				
$B)$ $n = 4,$ $\sigma = .01$ $\beta = .1$	$(4)\pi^s = 40.8$.	$(3)\pi^s$ $(1)\pi^m$				
$C)$ $n = 5,$ $\sigma = .001$ $\beta = .0001$	$(5)\pi^s = 2861$.	$(4)\pi^s = 876$ $(1)\pi^m = 2869$	$(3)\pi^s = 419$ $(2)\pi^m = 1371$	$(2)\pi^s = 244$ $(3)\pi^m = 800$	$(1)\pi^s = 160$ $(4)\pi^m = 524$	. $(5)\pi^m = 396$

T

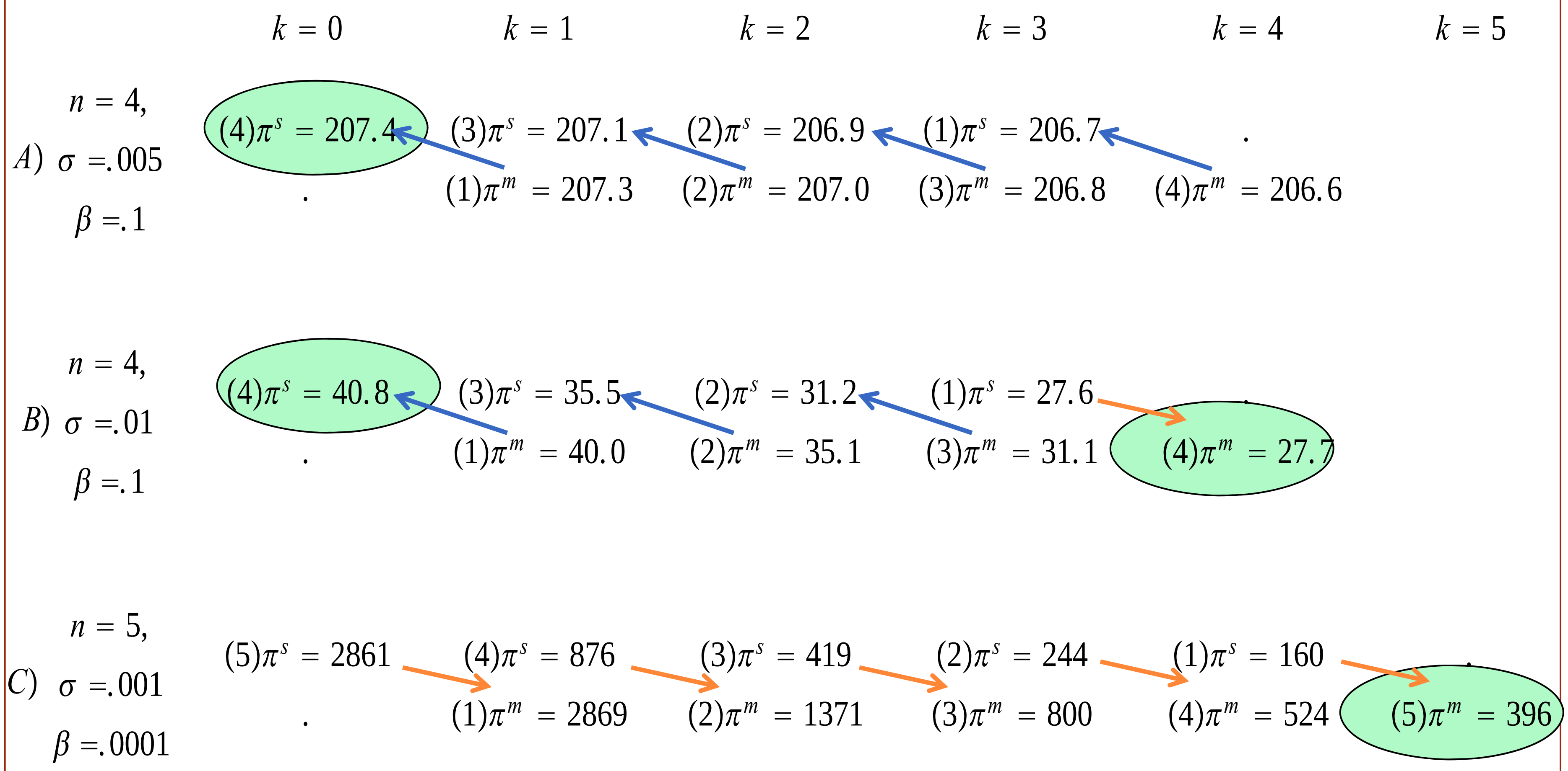
- Is th

Numerical s

Finally, note that

-as σ increases …(ceteris paribus)

-as n increases …



	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$A)$ $n=4,$ $\sigma=.005$ $\beta=.1$	$(4)\pi^s=207.4$.	$(3)\pi^s=207.1$ $(1)\pi^m=207.3$	$(2)\pi^s=206.9$ $(2)\pi^m=207.0$	$(1)\pi^s=206.7$ $(3)\pi^m=206.8$	. $(4)\pi^m=206.6$	
$B)$ $n=4,$ $\sigma=.01$ $\beta=.1$	$(4)\pi^s=40.8$.	$(3)\pi^s=35.5$ $(1)\pi^m=40.0$	$(2)\pi^s=31.2$ $(2)\pi^m=35.1$	$(1)\pi^s=27.6$ $(3)\pi^m=31.1$	. $(4)\pi^m=27.7$	
$C)$ $n=5,$ $\sigma=.001$ $\beta=.0001$	$(5)\pi^s=2861$.	$(4)\pi^s=876$ $(1)\pi^m=2869$	$(3)\pi^s=419$ $(2)\pi^m=1371$	$(2)\pi^s=244$ $(3)\pi^m=800$	$(1)\pi^s=160$ $(4)\pi^m=524$	. $(5)\pi^m=396$

Concluding comments

Concluding bullet-points:

- We have taken into account the possibility for firms of strategically choosing whether or not to consider the effect of their own production decision upon the market total output.
- This amounts to saying that firms can choose whether to behave in an oligopolistic or monopolistically competitive way.
- The textbook assumption of oligopoly *vs.* monopolistic competition is far from being an innocent assumption, if firms are allowed to make a choice concerning the consideration of reciprocal interdependency.

Concluding comments

- **When the number of firms is limited**, the explicit consideration of the interdependence turns out to be a dominant strategy, if the degree of product differentiation is high.
- For small levels of the degree of product differentiation, two equilibria may arise,
- The allocation in which all firms behave in a monopolistically competitive way is Pareto inefficient for the firms with respect to the oligopoly setting.

- The story becomes more involved in the presence of a large number of firms supplying a given market.

Concluding comments

- For a firm to find it convenient to be myopic in front of a population of smart rivals, the industry must be sufficiently fragmented (and other conditions must be met, connecting cost and demand parameters).
- For a firm to find it convenient to be smart in front of a population of myopic rivals, the industry must be not-highly fragmented (in relation with cost and demand parameters).
- (For given N and cost parameter), if parameter σ is large/small (entailing a low/high degree of product differentiation) the unilateral deviation from the fully myopic equilibrium is not-profitable/profitable.

Concluding comments

Possible simple extensions to our model:

- Heterogenous firms

  In the two-firm model, it is immediate to consider the situation in which c or β differ across firms

  This kind of heterogeneity does not lead to the stability of the mixed setting

  ➔Robustness

- Costly acquisition of information concerning Q

  Conclusions appear to be rather trivial

Concluding comments

Possible simple extensions to our model:

- Truly dynamic game

  In the present paper: static games and comparative statics across different settings

  (Evolution of firm’s behaviour over time – at least in a two period game)

Concluding comments

Provocative question:

- Heterogeneous behaviours are observed in some markets, in the real world
- (e.g., in market for some food goods)
- ➔ Is our theoretical model inconsistent with the real world?
  - No!...
    - These situation can be unstable (in transition).
    - An equilibrium may exist in mixed strategy (and the mixed strategy can be interpreted as a mixture of different behaviours observed ...).

Concluding comments

We remember that

- A recent literature line investigates markets in which O and MC firms co-exist (typically, big O firms and a fringe of atomistic MC firm).
  - But, the nature of the firms is exogenous
  - No possibility for firms to choose their nature; at most, 'divisionalization' for large firm(s))
  - (see, e.g., Anderson et al., 2013; Parenti, 2013; Shimomura and Thisse, 2014; Kokovin et al., 2014)
  - In this line, the nature of firm's behavior is not endogenous. …

Concluding comments

We remember that

- Chirco Colombo Scrimitore (2013, *Theory and Decision*):
  - Theoretical, Empirical and Experiment evidence supporting heterogeneous firms in one market
  - In their works, heterogeneity has to do with the final goal of firms, that is, profit maximization or rather relative profit maximization in the market stage.
  - Relative profit max is a commitment taken in a pre-market stage
  - Heterogeneity is possible under specific circumstances.

Thanks!

Roberto

cellini@unict.it
http://www.robertocellini.it